Statement 1

The following numbers are all prime numbers:

a) 1381;

b) 1567;

c) 48253;

d) 524287;

e) 170141183460469231731687303715884105727

Statement 2

The following formula always produces prime numbers:

$N^2 - N + 41$

For example, if N = 3 then

$N^2 - N + 41 = 3^2 - 3 + 41 = 47$

or if N = 10 then

$N^2 - N + 41 = 10^2 - 10 + 41 = 131.$

Statement 3

Every even number larger than 4 can be written as the sum of two prime numbers.

For example: 24 = 19 + 5 or 156 = 67 + 89.

Statement 4

Prime numbers go on for ever. In other words, there is no such thing as the last prime number.

Statement 5

For special cases the formula $2^N - 1$ produces prime numbers.

The values less than 258 which work are:

N = 1, 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257

For example when N = 5, $2^5 - 1 = 31$ which is prime,

but when N = 6, $2^6 - 1 = 63$ which is not prime.

Statement 6

The following formula always produces prime numbers:

$N^2 - 79N + 1601.$

For example, if N = 2 then $N^2 - 79N + 1601 =$

$2^2 - 79 \times 2 + 1601 = 1447$

or if N = 15 then $N^2 - 79N + 1601 =$

$15^2 - 79 \times 15 + 1601 = 641$

Statement 7

For special cases the formula $2^M + 1$ produces prime numbers.

The values for which the formula works are the powers of 2:

M = 1, 2, 4, 8, 16, 32, 64, 128, . . .

For example when M = 2

$2^2 + 1 = 5$ which is prime,

but when M = 3

$2^3 + 1 = 9$ which is not prime.

Statement 8

Every odd number greater than 3 can be written in the form:

2P + Q where P and Q are prime numbers.

For example:

27 = 2 x 11 + 5,

1057 = 2 x 487 + 83.

Statement 9

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Now, it is your turn!

Background 3:
In a letter dated June 7, 1742, Christian Goldbach claimed this result was true. He was writing to the mathematician Leonhard Euler. In the letter, he wrote about several ideas connected with prime numbers including the statement on this card. Goldbach was a professor in St Petersburg when Euler first met him and they corresponded about mathematics from 1729 to 1764.

In response to Goldbach's letter, Euler wrote back on June 30. He responded to Goldbach's idea. Now it is your turn to work out a response.

Background 2:
Throughout the history of mathematics people have always tried to simplify the subject by developing new methods. In the area of prime numbers, some have looked for a formula that would always produce prime numbers.

It is now much easier to investigate prime number questions because we have computers which speed up the calculations. Perhaps you could write a program or use a spreadsheet to work on this statement.

Background 1:
Throughout the history of mathematics people have investigated prime numbers. They have tried to find simple methods to check whether particular numbers are prime. They have also tried to find new prime numbers and become the world record holder. Other people have then checked if these new numbers are truly prime.

In 1978, Laura Nickel and Curt Noll, California high school students, used a supercomputer to find the world record prime number at the time. It was 6533 digits long. The following year they broke their own record, finding a prime number that was 6987 digits long. At the time of writing the largest known prime number is 65,050 digits long.

Background 6:
Throughout the history of mathematics people have always tried to simplify the subject by developing new methods. In the area of prime numbers people have looked for a formula that would always produce prime numbers.

It is now much easier to investigate prime number questions because we have computers which speed up the calculations. Perhaps you could write a program or use a spreadsheet to work on this statement.

Background 5:
The person who made this claim was Father Marin Mersenne (1558-1648). He was a Jesuit priest, as well as an able mathematician and physicist. His most important contribution to mathematics and science was that he brought scholars together to talk about their work. He also corresponded with numerous thinkers around Europe asking questions and suggesting answers which they then worked on.

He made this claim in 1644, obviously at a time long before calculators. When he died four years later it had not been challenged. What do you think, over 350 years later? Is it true?

Background 4:
This is one of the most basic questions which can be asked about prime numbers. As long ago as about 300 BC a mathematician called Euclid was working on this problem. He made his mind up about it and published his idea in a huge mathematical piece of work called "The Elements". It was organised into 13 books with over 450 results. Many people believe it to be the most influential piece of mathematics ever written.

What conclusion do you think he came to about prime numbers?

Background 9:
You will have realised by now that many people down through the ages have worked on prime numbers and have tried to come up with new truths. They have included full-time mathematicians and amateurs who study mathematics for pleasure. They have included teenagers and much older people. The history is a long one, but new results continue to be found.

Will you be the next?

Background 8:
This statement was thought to be true by Paul Levy (1886 - 1971). He made the claim in 1964 when he was 78. Sometimes it is thought that people do their best mathematics when they are young and that they can not do much creative work beyond the age of 50. However Levy, who is one of the creators of much modern probability theory, did his first major work in his forties.

How do you think he did on prime numbers? Is the statement true?

Background 7:
This claim for truth was made by the French mathematician Pierre de Fermat around 1640. Fermat lived from 1601 to 1665 and was not a full-time mathematician. He is sometimes called the "Prince of Amateurs". He was a lawyer by profession and had no special mathematical training. He became accepted as an important mathematician when Mersenne (see statement 5) invited him to present results to others in Paris.

He did much work in the area of number theory, including work on prime numbers, but he rarely wrote down why he believed something to be true. However, he would on occasions say that he had a proof, but that it was too long to write down. In this case he believed the result was true, but did not claim to have a proof.