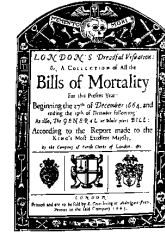
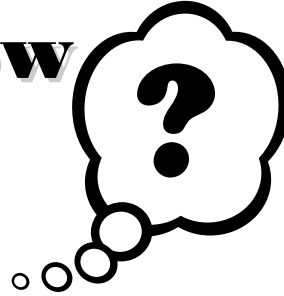


Do you know your vital statistics?



UNIT 2

In this unit students will use a range of real mortality statistics in order to cover areas of handling data and probability. At the same time it is hoped that students will be led to reflect on issues connected with life expectancy and death.

Using this unit

The unit is built around three main sets of statistics. They are:

- ◆ Life and death in England and Wales - 1992,
- ◆ John Graunt's London Life Table - 1662,
- ◆ Life table, 1990 to 1992.

Each source provides opportunity for statistical and probability work and provides a focus for discussion on some of the spiritual issues. Ideas are given in the teaching notes as to what might be the key discussion points at different times, but it is important to be flexible and to take opportunities as they arise.

Obviously, in tackling the area of death with students there is a need for sensitivity and it is important that students are encouraged to realise this from the outset. Teachers will need to consider whether it is an appropriate time to cover such a unit, particularly if a student has had a recent bereavement. Also, in using real statistics, there is a possibility that the 1992 data may have a personal significance to some students.

The work covered is suitable for Intermediate Level students.

Students will need to be able to:

- ◆ Calculate one quantity as a percentage of another;
- ◆ Work confidently with numbers up to millions;
- ◆ Round numbers to a given number of decimal places.

Mathematical content

AT4

- Analysing and interpreting data including
- ◆ Reading tables
 - ◆ Constructing frequency diagrams
 - ◆ Constructing cumulative frequency diagrams
 - ◆ Calculating averages

Estimating and calculating the probabilities of events including

- ◆ Using relative frequency to calculate probabilities
- ◆ Describing probability as a fraction, decimal and percentage
- ◆ Using standard probability notation

Spiritual and moral development

It is hoped that students will consider and reflect on how much they value their own life, what affects how long they live, how long they want to live and how they respond to and cope with death.

Background

The *Life and Death in England and Wales - 1992* and the *Life table - 1990 to 1992* sources are easily available in public libraries. The population figures are estimates based on the 1991 census. The death figures are based upon registered deaths recorded in England and Wales during 1992. The Life table, which can be found in the same document, is produced by the Government Actuary's Department and is based on the assumption that existing mortality rates continue. Each year, many other statistics connected with death rates and their causes are published by the Office of Population Censuses and Surveys.

John Graunt was considered to be the founder of demography. He was the first person to think of producing a life table based on mortality figures and, although many of his assumptions were wrong, it was a key step forward in the field. Others built upon his work including Edmund Halley, the astronomer, who published improved tables in 1693. One of the driving forces behind developing this field was the emerging life assurance business.

Additional sources

1992 - *Mortality Statistics, general, England and Wales* (OPCS, Series DH1, no. 27).

David M. Burton, *The History of Mathematics - an introduction* (Allyn and Bacon, 1985).

Notes on the activities

Life and death issues in 1992

Questions 1 to 5 (task 1)

The meaning of vital statistics and the first 5 questions could all be done orally with the whole class.

At this stage, students need the two "1992 Data Sheets", one on population, the other on deaths. These could be photocopied back to back.

Question 6 (task 1)

When using the population table, students may need to be helped with figures presented as 1,000's.

Questions 7 and 8 (task 1)

Some students will need help setting up the diagrams here and choosing appropriate scales.

Questions 10 and 11 (task 1)

These are to encourage the students to explore the data for themselves and at this stage they could also be encouraged to pose their own questions.

Task 1 answers:

- 1) 51, 276, 900
- 2) 558, 313

- | | | |
|----------------|-------------|-------------|
| 3) 1.09% | | |
| 4a) 567,400 | 4b) 839,600 | 4c) 326,800 |
| 5a) 154 | 5b) 1,709 | 5c) 15,174 |
| 6a) i) 340,400 | ii) 321,800 | iii) 60 |
| iv) 50 | | |
| 6b) i) 326,000 | ii) 326,100 | iii) 949 |
| iv) 666 | | |
| 6c) i) 38,600 | ii) 99,600 | iii) 6,317 |
| iv) 11,245 | | |
| 9) 0.09% | 0.03% | 0.06% |
| 0.09% | 0.21% | 0.61% |
| 1.80% | 4.44% | 10.36% |

Class discussion

After this initial activity is under way there should be opportunity to encourage discussion about some of the following:

- ◆ Is death something we feel comfortable talking about or is it a taboo subject?
- ◆ To what extent do we come into contact with death in modern society and how well do we cope with it?



Living and dying in London

The information on the 'John Graunt and his Bills of Mortality' sheet could be presented through teacher exposition. Whether the sheet or exposition is used it is important to check that students can use the life table to calculate probabilities.

Question 2 (task 2)

Students are required to know how to work out the probability of an event not happening.

Question 3 (task 2)

Students may need support in constructing the diagram.

Question 4 (task 2)

This is to encourage students to think about what probability can or cannot tell us.

Task 2 answers:

- | | | | |
|-------------|-------|--------------|-------|
| 1a) $16/25$ | 0.64 | 1b) $5/8$ | 0.625 |
| 1c) $16/25$ | 0.64 | 1d) $3/5$ | 0.6 |
| 1e) $1/3$ | 0.333 | | |
| 2a) $3/8$ | 0.375 | 2b) $19/25$ | 0.76 |
| 2c) $21/25$ | 0.84 | | |
| 4a) 3 or 4 | | 4b) 11 or 12 | |
| 4c) 26 | | | |

Class discussion

Having viewed the death rates in the seventeenth century, students could discuss:

- ◆ Whether considering death as a teenager has any relevance in the 20th century, given the modern survival rates;
- ◆ Whether having some knowledge of how long you may have to live affects the way you live your life;
- ◆ The unpredictability of life and the limitations of statistics.



How long have I got to live?

As a preliminary to this activity ask the students to consider what ages each one wants to live to and then to write it down.

Then find out what percentage want to live to at least the ages of

- a) 25 b) 35 c) 45 d) 55 e) 65 f) 75 g) 85

This needs to be done separately for males and females. These figures can then be compared with the figures in questions 1 and 2.

Also, record their expected life spans so that an average can be calculated for males and females. These figures can then be compared with the figures in question 4.

Question 4 (task 4)

Students may need support in interpreting the meaning of expectation of life at a given age.

Question 6 (task 4)

This again encourages students to consider the limitations of probability as a predictor of single events.

Task 4 answers:

- | | | | |
|------------|----------|------------|----------|
| 1a) 0.9923 | 99.2% | 1b) 0.9832 | 98.3% |
| 1c) 0.9665 | 96.7% | 1d) 0.9230 | 92.3% |
| 1e) 0.8023 | 80.2% | 1f) 0.5387 | 53.9% |
| 1g) 0.2002 | 20.0% | | |
| 2a) 0.9970 | 99.7% | 2b) 0.9925 | 99.3% |
| 2c) 0.9820 | 98.2% | 2d) 0.9540 | 95.4% |
| 2e) 0.8789 | 87.9% | 2f) 0.7031 | 70.3% |
| 2g) 0.3834 | 38.3% | | |
| 4a) 74.3 | 4b) 79.6 | 4c) 75.7 | 4d) 80.2 |

Class discussion

It would be good to end the unit by drawing various threads together.



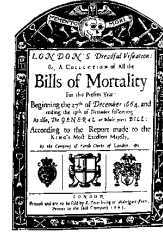
- ◆ Had they previously thought much about how long their life might be?
- ◆ Do we need to consider death more often?
- ◆ What other questions about life and death has the unit raised for them?

Combined probabilities (extension work)

It is possible to do a lot more analysis and calculation from the data provided. If the probability work needed to be extended students could work on combined survival rates. For example, they could consider the chance of a just-married couple of ages 30 and 25 surviving to their silver or golden wedding anniversary.

Also to extend students to a higher level they should be encouraged to explore the data themselves and ask their own questions.

Do you know your vital statistics?



UNIT 2

The proper meaning of the term 'vital statistics' is data that is connected with human life and aspects affecting it, such as the death rate. In this unit you will be using a variety of sources of data all concerned with life and death.

Life and death issues in 1992

1

Before using the first data sheet, write down your estimates for the following questions.

- In the year 1992, in England and Wales:
 1. What was the total population?
 2. How many people died?
 3. What percentage of the population died during the year?
 4. How many of the following age groups were living in the two countries:
 - a) 15 year olds b) 45 year olds c) 75 year olds?
 5. How many of the same age groups died during the year?
 - a) 15 year olds b) 45 year olds c) 75 year olds?
6. Now you will need the '1992 data sheet' to check your answers and do the following. Find out for the following ages:
 - a) 6 year olds b) 46 year olds c) 86 year olds
 - i) the number of males living in England and Wales in 1992;
 - ii) the number of females living in England and Wales;
 - iii) the number of males who died during the year;
 - iv) the number of females who died.
7. Draw a frequency diagram to show the population within the following age groups: 0 to 9, 10 to 19, 20 to 29, . . . 80 to 89.
8. Draw a frequency diagram to show the number of deaths using the same age groups as above.
9. Calculate the percentage of each of the above age groups that died during the year (answers accurate to 2 decimal places).
10. What differences do you notice between the male and female figures:
 - a) in the population table b) in the deaths table.
11. Write down any things that you have noticed from the data that surprised you.

Living and dying in London

An important part of the mathematics curriculum is Handling Data, which includes statistics and probability. There are two main ways that probability as we know it came to be developed. One is the study of gambling and games of chance and the other is in the field of insurance and mortality tables.

In this activity, you are going to learn about the early work on life and death statistics and then calculate the chances of living to certain ages.

2

First you need to read the sheet 'John Graunt and his Bills of Mortality'.

- From the London Life Table, work out the following probabilities, giving your answer as a fraction in its lowest terms and as a decimal accurate to three decimal places.
 - P (new baby surviving to 6)
 - P (6 year old surviving to 16)
 - P (26 year old surviving to 36)
 - P (46 year old surviving to 56)
 - P (66 year old surviving to 76)
- Work out the following probabilities giving answers in the same way:
 - P (6 year old not surviving to 16)
 - P (26 year old dying before becoming 56)
 - P (new baby not living to 36)
- Complete the table below and then draw a cumulative frequency curve to show the data

Age	0	6	16	26	36	46	56	66	76
Total % dead by the age	0	36	60						
- Use your graph to answer the following. By what age would you expect
 - 25%
 - 50%
 - 75%
 of the population to be dead.
- In 1669, Ludwig Huygens, a Dutch mathematician, had also been working on expected length of life. In a letter to his brother Christian, he wrote "I have just been making a table showing how long people of a given age have to live . . . Live well! according to my figures you will live to about 56.5 and I will live to 55."

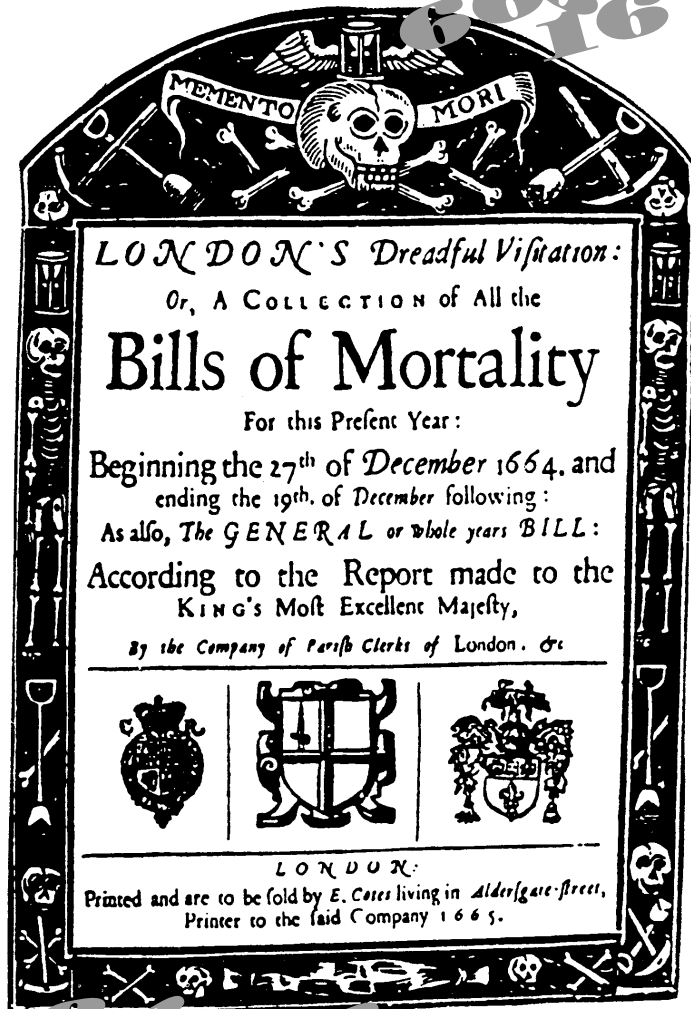
Write down whether you think Huygens was right in saying he could predict his and his brother's life spans. Explain your reasoning.

John Graunt and his Bills of Mortality

John Graunt was a London merchant who lived from 1620 to 1674. He is said to be the founder of the science of demography, that is, the study of human population. He was the first person to attempt to interpret data that had been collected from as far back as 1563. Each week the number of deaths in London parishes was recorded. The deaths were classified according to sex and cause. These records were summarised each year in what was known as the Bills of Mortality.

In 1662 Graunt published a tract called *Natural and Political Observation Made Upon the Bills of Mortality*. It was important enough for Graunt to be made a member of the Royal Society. He was the only shopkeeper amongst a group of very learned men. Graunt drew many conclusions from the data he worked on. One was that although more males were born than females, since women lived longer than men, things would even out and there would be no need for polygamy!

However, perhaps the most significant part of Graunt's work was the idea he had to turn the death rates into what he called a life table. Based on the data he analysed, he felt able to predict what would happen to 100 children born in the same year. So, for example, the table says that by the age of 16 only 40 would still be alive, by the age of 66 only 3 would be expected to be alive still. The life table could be used to calculate the probability that people would live to a certain age. For example:



$$P(16 \text{ year old living to } 26) = \frac{(\text{no. alive at } 26)}{(\text{no. alive at } 16)} = \frac{25}{40} \text{ or } 0.625$$

$$P(\text{new baby living to } 26) = \frac{25}{100} \text{ or } 0.25$$

$$P(26 \text{ year old living to } 46) = \frac{10}{25} \text{ or } 0.4$$

Age	Survivors
0	100
6	64
16	40
26	25
36	16
46	10
56	6
66	3
76	1

Although, the London Life Table had many flaws, it was still a key piece of work and laid the foundation for many others to develop the subject.

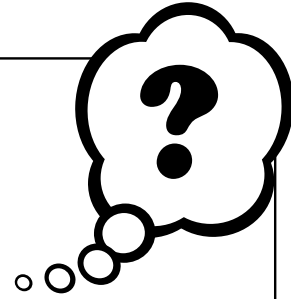
How long have I got to live?

Since the time of John Graunt, the statistics of life and death have come a long way. Each year new life tables are published which are based on the most up to date death rates (usually the last 3 years). In some ways they are very similar to Graunt's first life table, but they also include a new figure called life expectancy. This is how much longer somebody of a certain age might be expected to live, on average. For example, if someone of age 45 has a life expectation of 30 then they might expect to live to 75.

3

Before you work with a recent life table, stop for a moment and think again about how long you expect to live.

Also, write down the probability that you will die.



4

You now need the 'Life Table 1990 - 1992' sheet.

- From the life table for males, calculate the probability that a 15 year old boy survives to:
 - 25
 - 35
 - 45
 - 55
 - 65
 - 75
 - 85

Give your answers as a decimal accurate to four decimal places and as a percentage accurate to one decimal place.
- Do the same for a 15 year old girl.
- Compare these figures with the class figures you did earlier. Write down what you notice.
- Using the life expectation figures, work out the average age the following might expect to live to:
 - 15 year old boy
 - 15 year old girl
 - 45 year old man
 - 40 year old woman
- Compare 4a) and 4b) with the average age the males and females in your class hope to live to.
- Is it possible to use these tables to predict how long you will live?

If so, how? If not, why not?



Life Table, 1990 to 1992 England and Wales

(Supplied by the Government Actuary's Department)

Age x	Males		Females	
	l_x	e_x	l_x	e_x
0	10,000	73.4	10,000	79.0
1	9,919	73.0	9,937	78.4
2	9,912	72.1	9,931	77.5
3	9,909	71.1	9,928	76.5
4	9,906	70.1	9,926	75.5
5	9,903	69.1	9,924	74.5
10	9,894	64.2	9,917	69.6
15	9,883	59.3	9,910	64.6
20	9,850	54.4	9,896	59.7
25	9,807	49.7	9,880	54.8
30	9,765	44.9	9,862	49.9
35	9,717	40.1	9,836	45.0
40	9,649	35.4	9,795	40.2
45	9,552	30.7	9,732	35.5
50	9,393	26.2	9,625	30.8
55	9,122	21.8	9,454	26.3
60	8,671	17.8	9,171	22.1
65	7,929	14.3	8,710	18.1
70	6,804	11.2	7,997	14.5
75	5,324	8.6	6,968	11.2
80	3,614	6.4	5,561	8.4
85	1,979	4.8	3,799	6.1

Notes:

l_x is the number who would survive to exact age x out of 10,000 born, if they were subject throughout their lives to the recorded death rates of 1990-92.

e_x is the expectation of life, i.e., the average future lifetime which would be lived by a person of exact age x, if likewise subject to these death rates.

1992 Data Sheet - Population

1992 Data Sheet

Series DH1 no. 27 Table 1

Table 1 Estimated resident population (1991 Census based):
sex and age, as at 30 June 1992England and Wales
thousands

Age	Persons	Males	Females	Age	Persons	Males	Females
All ages	51,276.9	25,098.6	26,178.2				
0-4	3,454.6	1,771.5	1,683.1	50-54	2,722.9	1,360.9	1,362.0
0	694.5	355.9	338.6	50	559.7	280.4	279.3
1	700.7	358.8	341.9	51	516.0	258.0	258.0
2	686.4	352.2	334.2	52	546.9	273.6	273.4
3	682.6	350.1	332.5	53	552.2	275.4	276.8
4	690.5	354.5	336.0	54	548.1	273.7	274.5
5-9	3,258.9	1,674.0	1,584.9	55-59	2,577.0	1,281.8	1,295.2
5	669.8	343.8	326.0	55	536.5	268.1	268.4
6	662.3	340.4	321.8	56	527.2	262.4	264.7
7	659.2	338.5	320.7	57	516.0	257.3	258.6
8	632.5	324.8	307.8	58	499.2	247.9	251.3
9	635.0	326.5	308.6	59	498.1	246.0	252.1
10-14	3,126.6	1,607.6	1,518.9	60-64	2,533.6	1,228.1	1,305.5
10	632.9	325.6	307.2	60	510.2	252.0	258.2
11	643.5	330.1	313.5	61	518.8	253.3	265.6
12	649.0	333.4	315.5	62	513.0	247.3	265.7
13	625.4	322.1	303.4	63	500.1	240.2	259.9
14	575.8	296.4	279.3	64	491.4	235.3	256.1
15-19	3,106.5	1,598.6	1,508.0	65-69	2,422.5	1,129.3	1,293.2
15	567.4	292.5	274.8	65	492.4	234.5	257.9
16	592.3	305.1	287.2	66	494.6	234.7	259.9
17	618.1	318.2	299.9	67	483.3	225.3	258.0
18	641.1	329.3	311.8	68	478.5	220.1	258.3
19	687.7	353.5	334.2	69	473.7	214.6	259.1
20-24	3,878.9	1,984.4	1,894.5	70-74	2,115.1	919.9	1,195.2
20	732.9	376.7	356.2	70	485.3	218.0	267.3
21	773.2	395.4	377.8	71	495.7	218.3	277.4
22	768.7	394.6	374.1	72	503.4	218.5	284.9
23	798.8	407.5	391.3	73	327.1	138.9	188.2
24	805.3	410.1	395.1	74	303.6	126.2	177.4
25-29	4,255.6	2,168.8	2,086.8	75-79	1,625.4	638.1	987.3
25	830.6	423.2	407.4	75	326.8	134.1	192.6
26	848.6	432.3	416.3	76	336.9	135.7	201.2
27	865.2	441.1	424.1	77	339.4	132.4	206.9
28	864.4	441.0	423.5	78	323.8	123.8	200.0
29	846.7	431.2	415.5	79	298.6	112.1	186.5
30-34	3,881.5	1,967.8	1,913.8	80-84	1,151.4	389.8	761.6
30	826.2	420.2	406.1	80	272.4	98.8	173.6
31	805.1	410.2	394.9	81	250.3	87.8	162.5
32	767.7	390.1	377.5	82	230.2	77.5	152.7
33	749.1	377.7	371.4	83	209.8	67.6	142.3
34	733.6	369.6	363.9	84	188.6	58.0	130.6
35-39	3,407.5	1,711.8	1,695.6	85-89	598.2	159.6	438.6
35	710.3	357.5	352.8	85	163.3	47.6	115.7
36	684.5	344.1	340.3	86	138.3	38.6	99.6
37	664.4	333.8	330.5	87	117.7	30.8	86.9
38	675.5	339.0	336.5	88	98.6	24.1	74.5
39	672.9	337.4	335.4	89	80.3	18.5	61.9
40-44	3,516.5	1,760.5	1,756.1	90 and over	249.3	46.3	203.0
40	660.8	330.5	330.3				
41	673.9	338.2	335.7	All ages			
42	694.2	347.1	347.1	under 16	10,407.4	5,345.6	5,061.8
43	718.7	360.0	358.7	All ages			
44	769.0	384.6	384.3	under 18	11,617.8	5,968.9	5,649.8
45-49	3,394.9	1,700.0	1,594.9				
45	839.6	421.7	417.9	16-44	21,479.2	10,899.3	10,579.9
46	652.1	326.0	326.1				
47	643.4	321.5	321.9	45-64/59*	9,922.9	5,570.8	4,352.1
48	646.9	323.9	323.0				
49	612.9	306.9	306.1	65/60			
				and over**	9,467.3	3,282.9	6,184.4

* 45-64 for males; 45-59 for females

** 65 for males; 60 for females

Notes: 1. Figures may not add due to rounding

2. The population estimates at mid 1992 are based on the 1991 Census of Population. They are not fully comparable with those for earlier years, which were based on 1981 Census results with allowances for subsequent birth, death and migrations. The estimates for 1982-90 have been revised and are published in *OPCS Monitor*.

Series PP1 94/1

1992 Data Sheet - Deaths

1992 Data Sheet
no. 27 Table 3

Series DH1

table 3 deaths: age, sex and marital status, 1992

Note: Cases in which the age of the deceased was not definitely stated have been allocated to particular ages in accordance with the best available information and it is due regard to the circumstances and cause of death in each case.

Age last birthday	Persons	Males	Females	Ages last birthday	Persons	Males	Female
		Total	Total			Total	Total
All ages	558,313	271,732	286,581				
0-4	5,413	3,093	2,320	55-59	20,014	12,514	7,500
0	4,539	2,606	1,933	55	3,322	2,104	1,218
1	358	191	167	56	3,559	2,237	1,322
2	215	118	97	57	3,983	2,453	1,530
3	163	90	73	58	4,372	2,736	1,636
4	138	88	50	59	4,778	2,984	1,794
5-9	516	302	214	60-64	34,245	21,160	13,085
5	110	72	38	60	5,496	3,422	2,074
6	110	60	50	61	6,204	3,835	2,369
7	107	68	39	62	6,853	4,229	2,624
8	91	50	41	63	7,445	4,615	2,830
9	98	52	46	64	8,247	5,059	3,188
10-14	521	322	199	65-69	55,025	33,344	21,681
10	89	55	34	65	9,100	5,497	3,603
11	84	54	30	66	10,034	6,191	3,843
12	115	75	40	67	10,931	6,700	4,231
13	98	62	36	68	11,990	7,176	4,814
14	135	76	59	69	12,970	7,780	5,190
15-19	1,401	969	432	70-74	74,755	43,013	31,742
15	154	97	57	70	14,445	8,563	5,882
16	196	128	68	71	16,275	9,583	6,692
17	300	205	95	72	17,399	9,948	7,451
18	361	257	104	73	13,221	7,464	5,757
19	390	282	108	74	13,415	7,455	5,960
20-24	2,236	1,621	615	75-79	91,328	47,696	43,632
20	405	285	120	75	15,174	8,178	6,996
21	460	333	127	76	17,272	9,282	7,990
22	458	337	121	77	19,106	10,095	9,011
23	438	316	122	78	19,813	10,244	9,569
24	475	350	125	79	19,963	9,897	10,066
25-29	2,541	1,801	740	80-84	100,882	44,966	55,916
25	447	316	131	80	20,369	9,875	10,494
26	492	355	137	81	20,202	9,323	10,879
27	489	365	124	82	20,366	9,155	11,211
28	564	388	176	83	20,220	8,650	11,570
29	549	377	172	84	19,725	7,963	11,762
30-34	2,932	1,950	982	85-89	80,315	27,596	52,719
30	539	382	157	85	18,535	7,094	11,441
31	596	400	196	86	17,562	6,317	11,245
32	579	373	206	87	16,471	5,576	10,895
33	590	385	205	88	14,599	4,676	9,923
34	628	410	218	89	13,148	3,933	9,215
35-39	3,807	2,413	1,394	90-94	41,051	10,076	30,975
35	688	432	256	90	11,504	3,151	8,353
36	686	433	253	91	9,712	2,514	7,198
37	704	436	268	92	8,251	1,963	6,288
38	848	559	289	93	6,472	1,402	5,070
39	881	553	328	94	5,112	1,046	4,066
40-44	5,862	3,539	2,323	95-99	12,059	1,992	10,067
40	924	554	370	95	4,120	741	3,379
41	1,015	636	379	96	2,997	507	2,490
42	1,145	659	486	97	2,294	357	1,937
43	1,243	773	470	98	1,552	228	1,324
44	1,535	917	618	99	1,096	159	937
45-49	8,938	5,407	3,531	100-104	1,832	233	1,599
45	1,709	1,016	693	100	738	101	637
46	1,615	949	666	101	506	68	438
47	1,757	1,072	685	102	299	34	265
48	1,891	1,156	735	103	176	20	156
49	1,966	1,214	752	104	113	10	103
50-54	12,515	7,710	4,805	105-109	123	15	108
50	2,070	1,262	808	105	63	5	58
51	2,095	1,300	795	106	32	4	28
52	2,539	1,559	980	107	17	5	12
53	2,828	1,749	1,079	108	10	-	10
54	2,983	1,840	1,143	109	1	1	-
				110 and over	2		2

